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Chapter 5

THE MAGNETIZATION CURVE AND INDUCTANCE

5.1 INTRODUCTION

As shown in Chapters 2 and 4, the induction machine configuration is quite complex. So far we elucidated the subject of windings and their mmfs. With windings in slots, the mmf has (in three-phase or two-phase symmetric windings) a dominant wave and harmonics. The presence of slot openings on both sides of the airgap is bound to amplify (influence, at least) the mmf step harmonics. Many of them will be attenuated by rotor-cage-induced currents. To further complicate the picture, the magnetic saturation of the stator (rotor) teeth and back irons (cores or yokes) also influence the airgap flux distribution producing new harmonics.

Finally, the rotor eccentricity (static and/or dynamic) introduces new harmonics in the airgap field distribution.

In general, both stator and rotor currents produce a resultant field in the machine airgap and iron parts.

However, with respect to fundamental torque-producing airgap flux density, the situation does not change notably from zero rotor currents to rated rotor currents (rated torque) in most induction machines, as experience shows.

Thus it is only natural and practical to investigate, first, the airgap field fundamental with uniform equivalent airgap (slotting accounted through correction factors) as influenced by the magnetic saturation of stator and rotor teeth and back cores, for zero rotor currents.

This situation occurs in practice with the wound rotor winding kept open at standstill or with the squirrel cage rotor machine fed with symmetrical a.c. voltages in the stator and driven at mmf wave fundamental speed ($n_1 = f_1/p_1$).

As in this case the pure travelling mmf wave runs at rotor speed, no induced voltages occur in the rotor bars. The mmf space harmonics (step harmonics due to the slot placement of coils, and slot opening harmonics etc.) produce some losses in the rotor core and windings. They do not notably influence the fundamental airgap flux density and, thus, for this investigation, they may be neglected, only to be revisited in Chapter 11.

To calculate the airgap flux density distribution in the airgap, for zero rotor currents, a rather precise approach is the FEM. With FEM, the slot openings could be easily accounted for; however, the computation time is prohibitive for routine calculations or optimization design algorithms.

In what follows, we first introduce the Carter coefficient K_c to account for the slotting (slot openings) and the equivalent stack length in presence of radial ventilation channels. Then, based on magnetic circuit and flux laws, we calculate the dependence of stator mmf per pole F_{1m} on airgap flux density

accounting for magnetic saturation in the stator and rotor teeth and back cores, while accepting a pure sinusoidal distribution of both stator mmf F_{lm} and airgap flux density, B_{lg} .

The obtained dependence of $B_{lg}(F_{lm})$ is called the magnetization curve. Industrial experience shows that such standard methods, in modern, rather heavily saturated magnetic cores, produce notable errors in the magnetizing curves, at 100 to 130% rated voltage at ideal no load (zero rotor currents). The presence of heavy magnetic saturation effects such as airgap, teeth or back core flux density, flattening (or peaking), and the rough approximation of mmf calculations in the back irons are the main causes for these discrepancies.

Improved analytical methods have been proposed to produce satisfactory magnetization curves. One of them is presented here in extenso with some experimental validation.

Based on the magnetization curve, the magnetization inductance is defined and calculated.

Later the emf induced in the stator and rotor windings and the mutual stator/rotor inductances are calculated for the fundamental airgap flux density. This information prepares the ground to define the parameters of the equivalent circuit of the induction machine, that is, for the computation of performance for any voltage, frequency, and speed conditions.

5.2 EQUIVALENT AIRGAP TO ACCOUNT FOR SLOTTING

The actual flux path for zero rotor currents when current in phase A is maximum $i_A = I\sqrt{2}$ and $i_B = i_C = -I\sqrt{2}/2$, obtained through FEM, is shown in [Figure 5.1](#). [4]

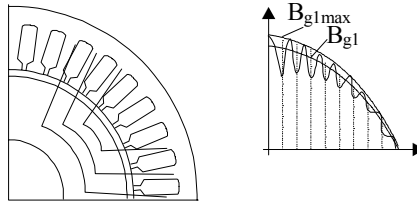


Figure 5.1 No-load flux plot by FEM when $i_B = i_C = -i_A/2$.

The corresponding radial airgap flux density is shown on [Figure 5.1b](#). In the absence of slotting and stator mmf harmonics, the airgap field is sinusoidal, with an amplitude of B_{g1max} .

In the presence of slot openings, the fundamental of airgap flux density is B_{g1} . The ratio of the two amplitudes is called the Carter coefficient.

$$K_C = \frac{B_{g1max}}{B_{g1}} \quad (5.1)$$

When the magnetic airgap is not heavily saturated, K_C may also be written as the ratio between smooth and slotted airgap magnetic permeances or between a larger equivalent airgap g_e and the actual airgap g .

$$K_C = \frac{g_e}{g} \geq 1 \quad (5.2)$$

FEM allows for the calculation of Carter coefficient from (5.1) when it is applied to smooth and double-slotted structure (Figure 5.1).

On the other hand, easy to handle analytical expressions of K_C , based on conformal transformation or flux tube methods, have been traditionally used, in the absence of saturation, though. First, the airgap is split in the middle and the two slottings are treated separately. Although many other formulas have been proposed, we still present Carter's formula as it is one of the best.

$$K_{C1,2} = \frac{\tau_{s,r}}{\tau_{s,r} - \gamma_{1,2} \cdot g / 2} \quad (5.3)$$

$\tau_{s,r}$ —stator/rotor slot pitch, g —the actual airgap, and

$$\gamma_{1,2} = \frac{4}{\pi} \left[\frac{b_{os,r}}{g} / \tan\left(\frac{b_{os,r}}{g}\right) - \ln \sqrt{1 + \left(\frac{b_{os,r}}{g}\right)^2} \right] \approx \frac{\left(2 \frac{b_{os,r}}{g}\right)^2}{5 + 2 \frac{b_{os,r}}{g}} \quad (5.4)$$

for $b_{os,r}/g \gg 1$. In general, $b_{os,r} \approx (3 - 8)g$. Where $b_{os,r}$ is the stator(rotor) slot opening.

With a good approximation, the total Carter coefficient for double slotting is

$$K_C = K_{C1} \cdot K_{C2} \quad (5.5)$$

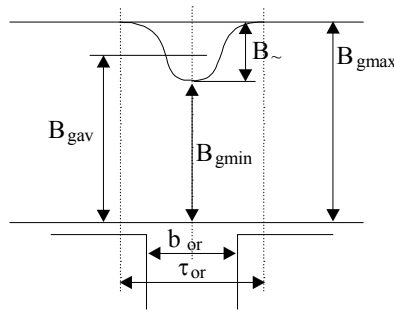


Figure 5.2 Airgap flux density for single slotting

The distribution of airgap flux density for single-sided slotting is shown on Figure 5.2. Again, the iron permeability is considered to be infinite. As the

magnetic circuit becomes heavily saturated, some of the flux lines touch the slot bottom (Figure 5.3) and the Carter coefficient formula has to be changed. [2] In such cases, however, we think that using FEM is the best solution.

If we introduce the relation

$$B_{\sim} = B_{g \max} - B_{g \min} = 2\beta B_{g \max} \quad (5.6)$$

the flux drop (Figure 5.2) due to slotting $\Delta\Phi$ is

$$\Delta\Phi_{s,r} = \sigma \frac{b_{os,r}}{2} B_{\sim s,r} \quad (5.7)$$

From [3],

$$\beta\sigma b_{os,r} = \frac{g}{2} \gamma_{1,2} \quad (5.8)$$

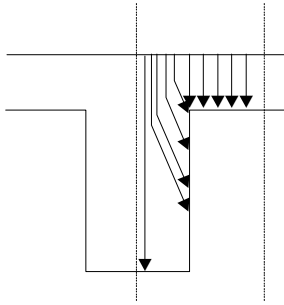


Figure 5.3 Flux lines in a saturated magnetic circuit

The two factors β and σ are shown on Figure 5.4 as obtained through conformal transformations. [3]

When single slotting is present, $g/2$ should be replaced by g .

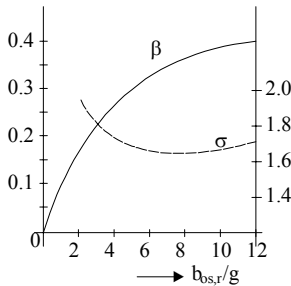


Figure 5.4 The factor β and σ as function of $b_{os,r}/(g/2)$

Another slot-like situation occurs in long stacks when radial channels are placed for cooling purposes. This problem is approached next.

5.3 EFFECTIVE STACK LENGTH

Actual stator and rotor stacks are not equal in length to avoid notable axial forces, should any axial displacement of rotor occurred. In general, the rotor stack is longer than the stator stack by a few airgaps (Figure 5.5).

$$l_r = l_s + (4 - 6)g \quad (5.9)$$

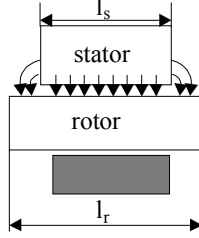


Figure 5.5. Single stack of stator and rotor

Flux fringing occurs at stator stack ends. This effect may be accounted for by apparently increasing the stator stack by $(2 \text{ to } 3)g$,

$$l_{se} = l_s + (2 \div 3)g \quad (5.10)$$

The average stack length, l_{av} , is thus

$$l_{av} \approx \frac{l_s + l_r}{2} \approx l_{se} \quad (5.11)$$

As the stacks are made of radial laminations insulated axially from each other through an enamel, the magnetic length of the stack L_e is

$$L_e = l_{av} \cdot K_{Fe} \quad (5.12)$$

The stacking factor K_{Fe} ($K_{Fe} = 0.9 - 0.95$ for $(0.35 - 0.5)$ mm thick laminations) takes into account the presence of nonmagnetic insulation between laminations.

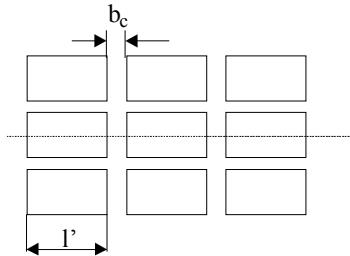


Figure 5.6 Multistack arrangement for radial cooling channels

When radial cooling channels (ducts) are used by dividing the stator into n elementary ones, the equivalent stator stack length L_e is (Figure 5.6)

$$L_e \approx (n+1)l'K_{Fe} + 2ng + 2g \quad (5.12)$$

$$\text{with} \quad b_c = 5-10 \text{ mm}; l' = 100-250 \text{ mm} \quad (5.13)$$

It should be noted that recently, with axial cooling, longer single stacks up to 500mm and more have been successfully built. Still, for induction motors in the MW power range, radial channels with radial cooling are in favor.

5.4 THE BASIC MAGNETIZATION CURVE

The dependence of airgap flux density fundamental B_{g1} on stator mmf fundamental amplitude F_{1m} for zero rotor currents is called the magnetization curve.

For mild levels of magnetic saturation, usually in general, purpose induction motors, the stator mmf fundamental produces a sinusoidal distribution of the flux density in the airgap (slotting is neglected). As shown later in this chapter by balancing the magnetic saturation of teeth and back cores, rather sinusoidal airgap flux density is maintained, even for very heavy saturation levels.

The basic magnetization curve ($F_{1m}(B_{g1})$ or $I_0(B_{g1})$ or I_0/I_n versus B_{g1}) is very important when designing an induction motor and notably influences the power factor and the core loss. Notice that I_0 and I_n are no load and full load stator phase currents and F_{1m0} is

$$F_{1m0} = \frac{3\sqrt{2}W_l K_{wl} I_0}{\pi p_l} \quad (5.14)$$

The no load (zero rotor current) design airgap flux density is $B_{g1} = 0.6 - 0.8T$ for 50 (60) Hz induction motors and goes down to 0.4 to 0.6 T for (400 to 1000) Hz high speed induction motors, to keep core loss within limits.

On the other hand, for 50 (60) Hz motors, I_0/I_n (no-load current/rated current) decreases with motor power from 0.5 to 0.8 (in subkW power range) to 0.2 to 0.3 in the high power range, but it increases with the number of pole pairs.

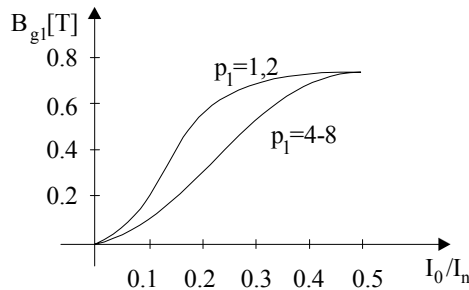


Figure 5.7 Typical magnetization curves

For low airgap flux densities, the no-load current tends to be smaller. A typical magnetization curve is shown in [Figure 5.7](#) for motors in the kW power range at 50 (60) Hz.

Now that we do have a general impression on the magnetising (mag.) curve, let us present a few analytical methods to calculate it.

5.4.1 The magnetization curve via the basic magnetic circuit

We shall examine first the flux lines corresponding to maximum flux density in the airgap and assume a sinusoidal variation of the latter along the pole pitch ([Figure 5.8a,b](#)).

$$B_{gl}(\theta_e, t) = B_{g1m} \cos(p_1\theta - \omega_1 t); \quad \theta_e = p_1\theta \quad (5.15)$$

$$\text{For } t = 0 \quad B_g(\theta, 0) = B_{g1m} \cos p_1\theta \quad (5.16)$$

The stator (rotor) back iron flux density $B_{cs,r}$ is

$$B_{cs,r} = \frac{1}{2h_{cs,r}} \int_0^\theta B_{gl}(\theta, t) d\theta \cdot \frac{D}{2} \quad (5.17)$$

where $h_{cs,r}$ is the back core height in the stator (rotor). For the flux line in [Figure 5.8a](#) ($\theta = 0$ to π/p_1),

$$B_{cs1}(\theta, t) = \frac{1}{2} \frac{2}{\pi} \frac{\tau}{h_{cs}} \cdot B_{g1m} \sin(p_1\theta - \omega_1 t); \quad \tau = \frac{\pi D}{2p_1} \quad (5.18)$$

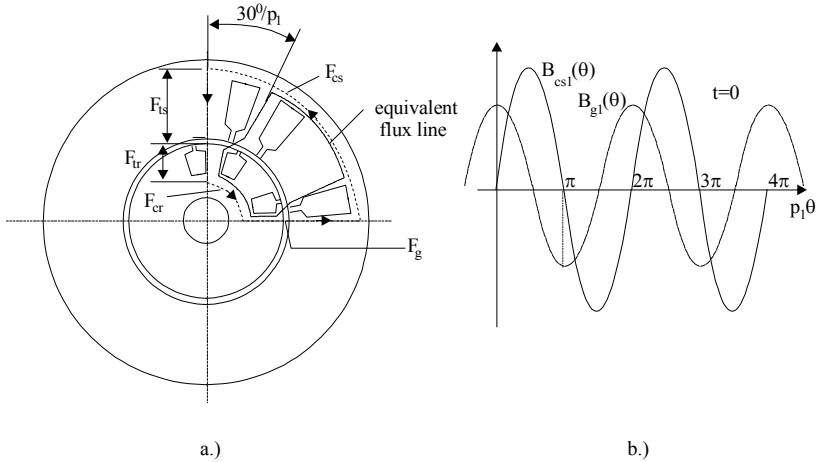


Figure 5.8 Flux path a.) and flux density types b.): ideal distribution in the airgap and stator core

Due to mmf and airgap flux density sinusoidal distribution along motor periphery, it is sufficient to analyse the mmf iron and airgap components F_{ts} , F_{tr} in teeth, F_g in the airgap, and F_{cs} , F_{cr} in the back cores. The total mmf is represented by F_{1m} (peak values).

$$2F_{1m} = 2F_g + 2F_{ts} + 2F_{tr} + F_{cs} + F_{cr} \quad (5.19)$$

Equation (5.19) reflects the application of the magnetic circuit (Ampere's) law along the flux line in [Figure 5.8a](#).

In industry, to account for the flattening of the airgap flux density due to teeth saturation, B_{g1m} is replaced by the actual (designed) maximum flattened flux density B_{gm} , at an angle $\theta = 30^\circ/p_1$, which makes the length of the flux lines in the back core 2/3 of their maximum length.

Then finally the calculated I_{1m} is multiplied by $2/\sqrt{3}$ ($1/\cos 30^\circ$) to find the maximum mmf fundamental.

At $\theta_{er} = p_1\theta = 30^\circ$, it is supposed that the flattened and sinusoidal flux density are equal to each other ([Figure 5.9](#)).

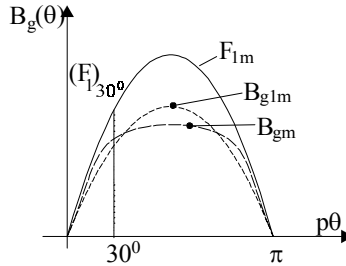


Figure 5.9 Sinusoidal and flat airgap flux density

We have to again write Ampere's law for this case (interior flux line in [Figure 5.8a](#)).

$$2(F_1)_{30^\circ} = 2F_g(B_{gm}) + 2F_{ts} + 2F_{tr} + F_{cs} + F_{cr} \quad (5.20)$$

and finally,

$$2F_{1m} = \frac{2(F_1)_{30^\circ}}{\cos 30^\circ} \quad (5.21)$$

For the sake of generality we will use (5.20) – (5.21), remembering that the length of average flux line in the back cores is 2/3 of its maximum.

Let us proceed directly with a numerical example by considering an induction motor with the geometry in [Figure 5.10](#).

$$\begin{aligned} 2p_1 &= 4; D = 0.1\text{m}; D_e = 0.176\text{m}; h_s = 0.025\text{m}; N_s = 24; \\ N_r &= 18; b_{r1} = 1.2b_{tr1}; b_{s1} = 1.4b_{ts1}; g = 0.5 \cdot 10^{-3}\text{m}; \\ h_r &= 0.018\text{m}; D_{\text{shaft}} = 0.035\text{m}; B_{gm} = 0.7\text{T} \end{aligned} \quad (5.22)$$

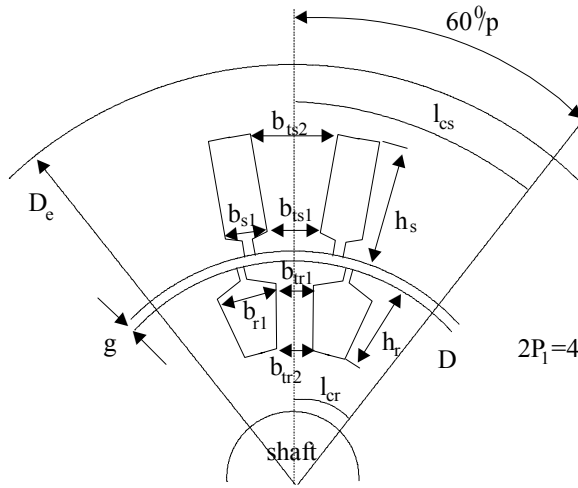


Figure 5.10 IM geometry for magnetization curve calculation

The B/H curve of the rotor and stator laminations is given in [Table 5.1](#).

Table 5.1 B/H curve a typical IM lamination

B[T]	0.0	0.05	0.1	0.15	0.2	0.25	0.3	0.35	0.4	0.45	0.5	0.55	0.6
H[A/m]	0	22.8	35	45	49	57	65	70	76	83	90	98	206
B[T]	0.65	0.7	0.75	0.8	0.85	0.9	0.95	1	1.05	1.1	1.15	1.2	1.25
H[A/m]	115	124	135	148	177	198	198	220	237	273	310	356	417
	1.3	1.35	1.4	1.45	1.5	1.55	1.6	1.65	1.7				
	482	585	760	1050	1340	1760	2460	3460	4800				
	1.75	1.8	1.85	1.9	1.95	2.0							
	6160	8270	11170	15220	22000	34000							

Based on (5.20) – (5.21), Gauss law, and B/H curve in [Table 5.1](#), let us calculate the value of F_{lm} .

To solve the problem in a rather simple way, we still assume a sinusoidal flux distribution in the back cores, based on the fundamental of the airgap flux density B_{g1m} .

$$B_{g1m} = \frac{B_{gm}}{\cos 30^\circ} = 0.7 \frac{2}{\sqrt{3}} = 0.809T \quad (5.23)$$

The maximum stator and rotor back core flux densities are obtained from (5.18):

$$B_{csm} = \frac{1}{\pi} \cdot \frac{\pi D}{2p_1} \frac{1}{h_{cs}} B_{g1m} \quad (5.24)$$

$$B_{\text{crm}} = \frac{1}{\pi} \cdot \frac{\pi D}{2p_1} \frac{1}{h_{\text{cr}}} B_{\text{g1m}} \quad (5.25)$$

$$\text{with } h_{\text{cs}} = \frac{(D_{\text{e}} - D - 2h_{\text{s}})}{2} = \frac{0.174 - 0.100 - 2 \cdot 0.025}{2} = 0.013\text{m} \quad (5.26)$$

$$h_{\text{cr}} = \frac{(D - 2g - D_{\text{shaft}} - 2h_{\text{r}})}{2} = \frac{0.100 - 0.001 - 2 \cdot 0.018 - 0.036}{2} = 0.0135\text{m}$$

Now from (5.24) – (5.25),

$$B_{\text{csm}} = \frac{0.1 \cdot 0.809}{4 \cdot 0.013} = 1.555\text{T} \quad (5.28)$$

$$B_{\text{crm}} = \frac{0.1 \cdot 0.809}{4 \cdot 0.0135} = 1.498\text{T} \quad (5.29)$$

As the core flux density varies from the maximum value cosinusoidally, we may calculate an average value of three points, say B_{csm} , $B_{\text{csm}}\cos 60^\circ$ and $B_{\text{csm}}\cos 30^\circ$:

$$B_{\text{csav}} = B_{\text{csm}} \left(\frac{1 + 4\cos 60^\circ + \cos 30^\circ}{6} \right) = 1.555 \cdot 0.8266 = 1.285\text{T} \quad (5.30)$$

$$B_{\text{crav}} = B_{\text{crm}} \left(\frac{1 + 4\cos 60^\circ + \cos 30^\circ}{6} \right) = 1.498 \cdot 0.8266 = 1.238\text{T} \quad (5.31)$$

From [Table 5.1](#) we obtain the magnetic fields corresponding to above flux densities. Finally, $H_{\text{csav}}(1.285) = 460 \text{ A/m}$ and $H_{\text{crav}}(1.238) = 400 \text{ A/m}$.

Now the average length of flux lines in the two back irons are

$$l_{\text{csav}} \approx \frac{2}{3} \cdot \frac{\pi(D_{\text{e}} - h_{\text{cs}})}{2p_1} = \frac{2}{3} \pi \frac{(0.176 - 0.013)}{4} = 0.0853\text{m} \quad (5.32)$$

$$l_{\text{crav}} \approx \frac{2}{3} \cdot \frac{\pi(D_{\text{shaft}} + h_{\text{cr}})}{2p_1} = \frac{2}{3} \pi \frac{(0.36 + 0.135)}{4} = 0.02593\text{m} \quad (5.33)$$

Consequently, the back core mmfs are

$$F_{\text{cs}} = l_{\text{csav}} \cdot H_{\text{csav}} = 0.0853 \cdot 460 = 39.238\text{Aturns} \quad (5.34)$$

$$F_{\text{cr}} = l_{\text{crav}} \cdot H_{\text{crav}} = 0.0259 \cdot 400 = 10.362\text{Aturns} \quad (5.35)$$

The airgap mmf F_{g} is straightforward.

$$2F_g = 2g \cdot \frac{B_{gm}}{\mu_0} = 2 \cdot 0.5 \cdot 10^{-3} \cdot \frac{0.7}{1.256 \cdot 10^{-6}} = 557.324 \text{Aturns} \quad (5.36)$$

Assuming that all the airgap flux per slot pitch traverses the stator and rotor teeth, we have

$$B_{gm} \cdot \frac{\pi D}{N_{s,r}} = (B_{ts,r})_{av} \cdot (b_{ts,r})_{av}; \quad b_{ts,r} = \frac{\pi(D \pm h_{s,r})}{N_{s,r}(1 + b_{s,r}/b_{ts,r})} \quad (5.37)$$

Considering that the teeth flux is conserved (it is purely radial), we may calculate the flux density at the tooth bottom and top as we know the average tooth flux density for the average tooth width $(b_{ts,r})_{av}$. An average can be applied here again. For our case, let us consider $(B_{ts,r})_{av}$ all over the teeth height to obtain

$$\begin{aligned} (b_{ts})_{av} &= \frac{\pi(0.1 + 0.025)}{(1 + 1.4) \cdot 24} = 6.81 \cdot 10^{-3} \text{ m} \\ (b_{tr})_{av} &= \frac{\pi(0.1 - 0.018)}{(1 + 1.2) \cdot 18} = 6.00 \cdot 10^{-3} \text{ m} \end{aligned} \quad (5.38)$$

$$\begin{aligned} (B_{ts})_{av} &= \frac{0.7 \cdot \pi \cdot 100}{24 \cdot 7.43 \cdot 10^{-3}} = 1.344 \text{ T} \\ (B_{tr})_{av} &= \frac{0.7 \cdot \pi \cdot 100}{18 \cdot 6.56 \cdot 10^{-3}} = 1.878 \text{ T} \end{aligned} \quad (5.39)$$

From Table 5.1, the corresponding values of $H_{tsav}(B_{tsav})$ and $H_{trav}(B_{trav})$ are found to be $H_{tsav} = 520 \text{ A/m}$, $H_{trav} = 13,600 \text{ A/m}$. Now the teeth mmfs are

$$2F_{ts} \approx H_{tsav}(2h_s) = 520 \cdot (0.025 \cdot 2) = 26 \text{Aturns} \quad (5.40)$$

$$2F_{tr} \approx H_{trav}(2h_r) = 13600 \cdot (0.018 \cdot 2) = 489.6 \text{Aturns} \quad (5.41)$$

The total stator mmf $(F_1)_{30^0}$ is calculated from (5.20)

$$2(F_1)_{30^0} = 557.324 + 26 + 489.6 + 39.238 + 10.362 = 1122.56 \text{Aturns} \quad (5.42)$$

The mmf amplitude F_{m10} (from 5.21) is

$$2F_{1m0} = \frac{2(F_1)_{30^0}}{\cos 30^0} = 1122.56 \cdot \frac{2}{\sqrt{3}} = 1297.764 \text{Aturns} \quad (5.43)$$

Based on (5.14), the no-load current may be calculated with the number of turns/phase W_1 and the stator winding factor K_{w1} already known.

Varying as the value of B_{gm} desired the magnetization curve— $B_{gm}(F_{1m})$ —is obtained.

Before leaving this subject let us remember the numerous approximations we operated with and define two partial and one equivalent saturation factor as K_{st} , K_{sc} , K_s .

$$K_{st} = 1 + \frac{2(F_{ts} + F_{tr})}{2F_g}; \quad K_{sc} = 1 + \frac{(F_{cs} + F_{cr})}{2F_g} \quad (5.44)$$

$$K_s = \frac{(F_l)_{30^\circ}}{2F_g} = K_{st} + K_{sc} - 1 \quad (5.45)$$

The total saturation factor K_s accounts for all iron mmfs as divided by the airgap mmf. Consequently, we may consider the presence of iron as an increased airgap g_{es} .

$$g_{es} = gK_cK_s; \quad K_s = K_s \left(\frac{I_0}{I_n} \right) \quad (5.46)$$

Let us notice that in our case,

$$K_{st} = 1 + \frac{26 + 489.6}{557.324} = 1.925; \quad K_{ct} = 1 + \frac{39.23 + 10.362}{557.324} = 1.089 \quad (5.47)$$

$$K_s = 1.925 + 1.089 - 1 = 2.103$$

A few remarks are in order.

- The teeth saturation factor K_{st} is notable while the core saturation factor is low; so the tooth are much more saturated (especially in the rotor, in our case); as shown later in this chapter, this is consistent with the flattened airgap flux density.
- In a rather proper design, the teeth and core saturation factors K_{st} and K_{sc} are close to each other: $K_{st} \approx K_{sc}$; in this case both the airgap and core flux densities remain rather sinusoidal even if rather high levels of saturation are encountered.
- In 2 pole machines, however, K_{sc} tends to be higher than K_{st} as the back core height tends to be large (large pole pitch) and its reduction in size is required to reduce motor weight.
- In mildly saturated IMs, the total saturation factor is smaller than in our case: $K_s = 1.3 - 1.6$.

Based on the above theory, iterative methods, to obtain the airgap flux density distribution and its departure from a sinusoid (for a sinusoidal core flux density), have been recently introduced [2,4]. However, the radial flux density components in the back cores are still neglected.

5.4.2 Teeth defluxing by slots

So far we did assume that all the flux per slot pitch goes radially through the teeth. Especially with heavily saturated teeth, a good part of magnetic path passes through the slot itself. Thus, the tooth is slightly “discharged” of flux.

We may consider that the following are approximates:

$$B_t = B_{ti} - c_1 B_g b_{s,r} / b_{ts,r}; \quad c_1 \ll 1.0 \quad (5.48)$$

$$B_{ti} = B_g \frac{b_{ts,r} + b_{s,r}}{b_{ts,r}} \quad (5.49)$$

The coefficient c_1 is, in general, adopted from experience but it is strongly dependent on the flux density in the teeth B_{ti} and the slotting geometry (including slot depth [2]).

5.4.3 Third harmonic flux modulation due to saturation

As only inferred above, heavy saturation in stator (rotor) teeth and/or back cores tends to flatten or peak, respectively, the airgap flux distribution.

This proposition can be demonstrated by noting that the back core flux density $B_{cs,r}$ is related to airgap (implicitly teeth) flux density by the equation

$$B_{cs,r} = C_{s,r} \int_0^{\theta_{er}} B_{ts,r}(\theta_{er}) d\theta_{er}; \quad \theta_{er} = p_1 \theta \quad (5.50)$$

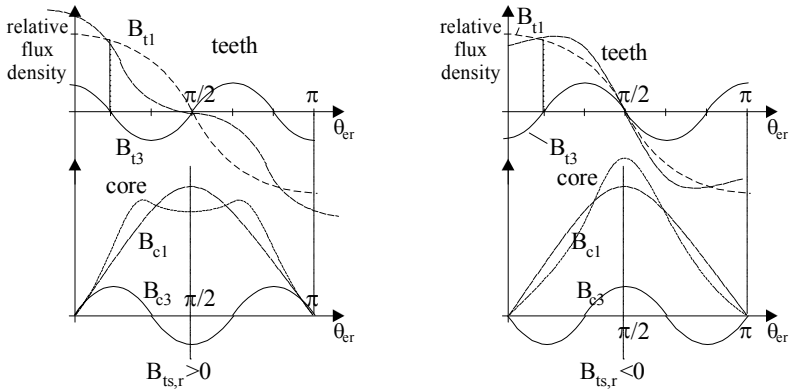


Figure 5.11 Tooth and core flux density distribution
a.) saturated back core ($B_{ts,r3} > 0$); b.) saturated teeth ($B_{ts,r3} < 0$)

Magnetic saturation in the teeth means flattening $B_{ts,r}(\theta)$ curve.

$$B_{ts,r}(\theta) = B_{ts,r1} \cos(\theta_{er} - \omega_1 t) + B_{ts,r3} \cos(3\theta_{er} - \omega_1 t) \quad (5.51)$$

Consequently, $B_{ts,r3} > 0$ means unsaturated teeth (peaked flux density, Figure 5.11a). With (5.51), equation (5.50) becomes

$$B_{cs,r} = C_{s,r} \left[B_{ts,r1} \sin(\theta_{er} - \omega_1 t) + \frac{B_{ts,r3}}{3} \sin(3\theta_{er} - \omega_1 t) \right] \quad (5.52)$$

Analyzing Figure 5.11, based on (5.51) – (5.52), leads to remarks such as

- Oversaturation of a domain (teeth or core) means flattened flux density in that domain (Figure 5.11b).
- In paragraph 5.4.1. we have considered flattened airgap flux density—that is also flattened tooth flux density—and thus oversaturated teeth is the case treated.
- The flattened flux density in the teeth (Figure 5.11b) leads to only a slightly peaked core flux density as the denominator 3 occurs in the second term of (5.52).
- On the contrary, a peaked teeth flux density (Figure 5.11a) leads to a flat core density. The back core is now oversaturated.
- We should also mention that the phase connection is important in third harmonic flux modulation. For sinusoidal voltage supply and delta connection, the third harmonic of flux (and its induced voltage) cannot exist, while it can for star connection. This phenomenon will also have consequences in the phase current waveforms for the two connections. Finally, the saturation produced third and other harmonics influence, notably the core loss in the machine. This aspect will be discussed in Chapter 11 dedicated to losses.

After describing some aspects of saturation – caused distribution modulation, let us present a more complete analytical nonlinear field model, which also allows for the calculation of actual spatial flux density distribution in the airgap, though with smoothed airgap.

5.4.4 The analytical iterative model (AIM)

Let us remind here that essentially only FEM [5] or extended magnetic circuit methods (EMCM) [6] are able to produce a rather fully realistic field distribution in the induction machine. However, they do so with large computation efforts and may be used for design refinements rather than for preliminary or direct optimization design algorithms.

A fast analytical iterative (nonlinear) model (AIM) [7] is introduced here for preliminary or optimization design uses.

The following assumptions are introduced: only the fundamental of m.m.f. distribution is considered; the stator and rotor currents are symmetric; - the IM cross-section is divided into five circular domains (Figure 5.12) with unique (but adjustable) magnetic permeabilities essentially distinct along radial (r) and tangential (θ) directions: μ_r , and μ_θ ; the magnetic vector potential A lays along the shaft direction and thus the model is two-dimensional; furthermore, the separation of variables is performed.

Magnetic potential, A, solution

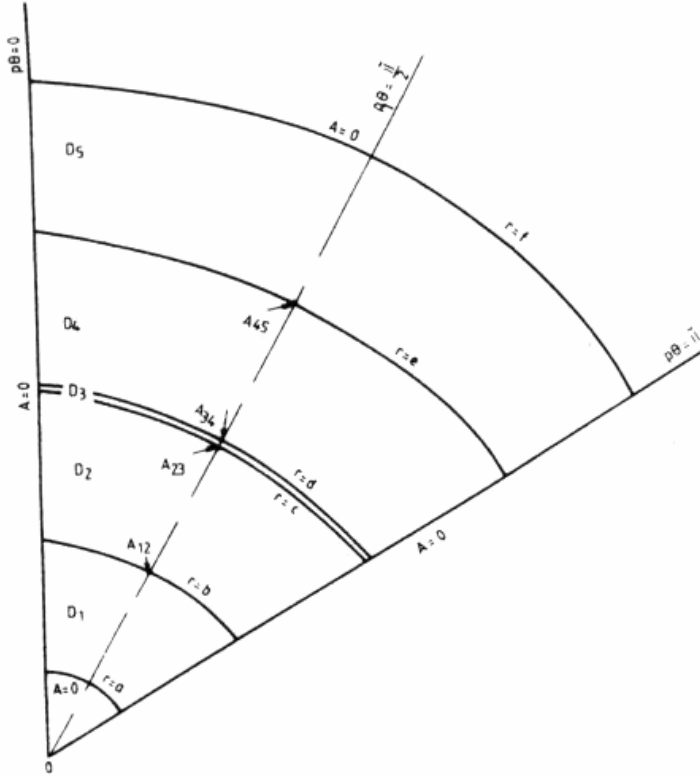


Figure 5.12 The IM cross-section divided into five domains

The Poisson equation in polar coordinates for magnetic potential A writes

$$\frac{1}{\mu_0} \left(\frac{1}{r} \frac{\partial A}{\partial r} + \frac{\partial^2 A}{\partial r^2} \right) + \frac{1}{\mu_r} \frac{1}{r^2} \frac{\partial^2 A}{\partial \theta^2} = -J \quad (5.53)$$

Separating the variables, we obtain

$$A(r, \theta) = R(r) \cdot T(\theta) \quad (5.54)$$

Now, for the domains with zero current (D_1, D_3, D_5) – $J = 0$, Equation (5.53) with (5.54) yields

$$\frac{1}{R(r)} \left(r \frac{dR(r)}{dr} + r^2 \frac{d^2 R(r)}{dr^2} \right) = \lambda^2; \quad \frac{\alpha^2}{T(\theta)} \frac{d^2 T(\theta)}{d\theta^2} = -\lambda^2 \quad (5.55)$$

with $\alpha^2 = \mu_\theta/\mu_r$ and λ a constant.

Also a harmonic distribution along θ direction was assumed. From (5.55):

$$r^2 \frac{d^2 R(r)}{dr^2} + r \frac{dR(r)}{dr} - \lambda^2 R = 0 \quad (5.56)$$

and

$$\frac{d^2 T(\theta)}{d\theta^2} + \frac{\lambda^2}{\alpha^2} T = 0 \quad (5.57)$$

The solutions of (5.56) and (5.57) are of the form

$$R(r) = C_1 r^\lambda + C_2 r^{-\lambda} \quad (5.58)$$

$$T(\theta) = C_3 \cos\left(\frac{\lambda}{\alpha} \theta\right) + C_4 \sin\left(\frac{\lambda}{\alpha} \theta\right) \quad (5.59)$$

as long as $r \neq 0$.

Assuming further symmetric windings and currents, the magnetic potential is an aperiodic function and thus,

$$A(r, 0) = 0 \quad (5.60)$$

$$A\left(r, \frac{\pi}{p_1}\right) = 0 \quad (5.61)$$

Consequently, from (5.59), (5.57) and (5.53), $A(r, \theta)$ is

$$A(r, \theta) = (g \cdot r^{p_1 \alpha} + h \cdot r^{-p_1 \alpha}) \sin(p_1 \theta) \quad (5.62)$$

Now, if the domain contains a homogenous current density J ,

$$J = J_m \sin(p_1 \theta) \quad (5.63)$$

the particular solution $A_p(r, \theta)$ of (5.54) is:

$$A_p(r, \theta) = K r^2 \sin(p_1 \theta) \quad (5.64)$$

with

$$K = -\frac{\mu_r \mu_\theta}{4\mu_r - p_1^2 \mu_\theta} J_m \quad (5.65)$$

Finally, the general solution of A (5.53) is

$$A(r, \theta) = (g \cdot r^{p_1 \alpha} + h \cdot r^{-p_1 \alpha} + K r^2) \sin(p_1 \theta) \quad (5.66)$$

As (5.66) is valid for homogenous media, we have to homogenize the slotting domains D_2 and D_4 , as the rotor and stator yokes (D_1 , D_5) and the airgap (D_3) are homogenous.

Homogenizing the Slotting Domains.

The main practical slot geometries (Figure 5.13) are defined by equivalent center angles θ_s , and θ_t , for an equivalent (defined) radius r_{m4} , (for the stator) and r_{m2} (for the rotor). Assuming that the radial magnetic field H is constant along the circles r_{m2} and r_{m4} , the flux linkage equivalence between the homogenized and slotting areas yields

$$\mu_t H \theta_t r_x L_1 + \mu_0 H \theta_s r_x L_1 = \mu_t H (\theta_t + \theta_s) r_x L_1 \quad (5.67)$$

Consequently, the equivalent radial permeability μ_r , is

$$\mu_r = \frac{\mu_t \theta_t + \mu_0 \theta_s}{\theta_t + \theta_s} \quad (5.68)$$

For the tangential field, the magnetic voltage relationship (along A, B, C trajectory on Figure 5.13), is

$$V_{mAC} = V_{mAB} + V_{mBC} \quad (5.69)$$

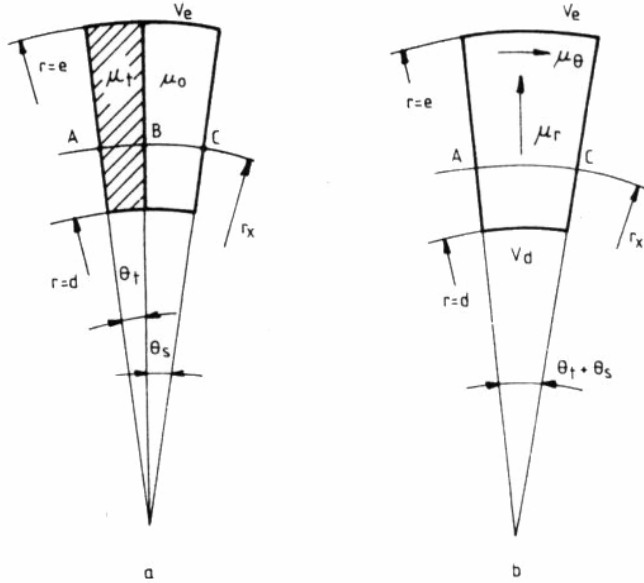


Figure 5.13 Stator and rotor slotting

With B_0 the same, we obtain

$$\frac{B_0}{\mu_0} (\theta_t + \theta_s) r_x = \frac{B_0}{\mu_t} \theta_t r_x + \frac{B_0}{\mu_0} \theta_s r_x \quad (5.70)$$

Consequently,

$$\mu_{\theta} = \frac{\mu_t \mu_0 (\theta_t + \theta_s)}{\mu_0 \theta_t + \mu_t \theta_s} \quad (5.71)$$

Thus the slotting domains are homogenized to be characterized by distinct permeabilities μ_r , and μ_{θ} along the radial and tangential directions, respectively.

We may now summarize the magnetic potential expressions for the five domains:

$$\begin{aligned} A_1(r, \theta) &= (g_1 r^{p_1} + h_1 r^{-p_1}) \sin(p_1 \theta); \quad a' = \frac{a}{2} < r < b \\ A_3(r, \theta) &= (g_3 r^{p_1} + h_3 r^{-p_1}) \sin(p_1 \theta); \quad c < r < d \\ A_5(r, \theta) &= (g_5 r^{p_1} + h_5 r^{-p_1}) \sin(p_1 \theta); \quad e < r < f \\ A_2(r, \theta) &= (g_2 r^{p_1 \alpha_2} + h_2 r^{-p_1 \alpha_2} + k_2 r^2) \sin(p_1 \theta); \quad b < r < c \\ A_4(r, \theta) &= (g_4 r^{p_1 \alpha_4} + h_4 r^{-p_1 \alpha_4} + k_4 r^2) \sin(p_1 \theta); \quad d < r < e \end{aligned} \quad (5.72)$$

with

$$\begin{aligned} K_4 &= -\frac{\mu_{r4} \mu_{\theta 4}}{4 \mu_{r4}^2 - p_1^2 \mu_{\theta 4}} J_{m4} \\ K_2 &= +\frac{\mu_{r2} \mu_{\theta 2}}{4 \mu_{r2}^2 - p_1^2 \mu_{\theta 2}} J_{m2} \end{aligned} \quad (5.73)$$

J_{m2} represents the equivalent demagnetising rotor equivalent current density which justifies the $\oplus$ sign in the second equation of (5.73). From geometrical considerations, J_{m2} and J_{m4} are related to the reactive stator and rotor phase currents I_{1s} and I_{2r}' , by the expressions (for the three-phase motor),

$$\begin{aligned} J_{m2} &= \frac{6\sqrt{2}}{\pi} \frac{1}{c^2 - b^2} W_1 K_{w1} I'_{2r} \\ J_{m4} &= \frac{6\sqrt{2}}{\pi} \frac{1}{e^2 - d^2} W_1 K_{w1} I_{1s} \end{aligned} \quad (5.74)$$

The main pole-flux-linkage Ψ_{m1} is obtained through the line integral of A_3 around a pole contour Γ (L_1 , the stack length):

$$\Psi_{m1} = \oint_{\Gamma} \bar{A}_3 d\bar{l} = 2L_1 A_3 \left(d, \frac{\pi}{2p_1} \right) \quad (5.75)$$

Notice that $A_3(d, \pi/2p_1) = -A_3(d, -\pi/2p_1)$ because of symmetry. With A_3 from (5.72), Ψ_{m1} becomes

$$\Psi_{m1} = 2L_1 (g_3 d^{p_1} + h_3 d^{-p_1}) \quad (5.76)$$

Finally, the e.m.f. E_1 , (RMS value) is

$$E_l = \pi\sqrt{2}f_l W_l K_{wl} \Psi_{ml} \quad (5.77)$$

Now from boundary conditions, the integration constants g_i and h_i are calculated as shown in the appendix of [7].

The computer program

To prepare the computer program, we have to specify a few very important details. First, instead of a (the shaft radius), the first domain starts at $a'=a/2$ to account for the shaft field for the case when the rotor laminations are placed directly on the shaft. Further, each domain is characterized by an equivalent (but adjustable) magnetic permeability. Here we define it. For the rotor and stator domains D_1 , and D_5 , the equivalent permeability would correspond to $r_{m1} = (a+b)/2$ and $r_{m5} = (e+f)/2$, and tangential flux density (and $\theta_0 = \pi/4$).

$$\begin{aligned} B_{1\theta}(r_{m1}, \theta_0) &= -p_1 \left[g_1 \left(\frac{a+b}{2} \right)^{p_1-1} - h_1 \left(\frac{a+b}{2} \right)^{-p_1-1} \right] \sin \frac{\pi}{4} \\ B_{5\theta}(r_{m5}, \theta_0) &= -p_1 \left[g_5 \left(\frac{e+f}{2} \right)^{p_1-1} - h_5 \left(\frac{e+f}{2} \right)^{-p_1-1} \right] \sin \frac{\pi}{4} \end{aligned} \quad (5.78)$$

For the slotting domains D_2 and D_4 , the equivalent magnetic permeabilities correspond to the radiuses r_{m2} and r_{m4} (Figure 5.13) and the radial flux densities B_{2r} , and B_{4r} .

$$\begin{aligned} B_{2r}(r_{m2}, \theta_0) &= p_1 \left(g_2 r_{m2}^{p_1\alpha_2-1} + h_2 r_{m2}^{-p_1\alpha_2-1} + K_2 r_{m2} \right) \cos p_1 \theta_0 \\ B_{4r}(r_{m4}, \theta_0) &= p_1 \left(g_4 r_{m4}^{p_1\alpha_4-1} + h_4 r_{m4}^{-p_1\alpha_4-1} + K_4 r_{m4} \right) \cos p_1 \theta_0 \end{aligned} \quad (5.79)$$

with $\cos p_1 \theta_0 = 0.9 \dots 0.95$. Now to keep track of the actual saturation level, the actual tooth flux densities B_{4t} , and B_{2t} , corresponding to B_{2r} , H_{2r} and B_{4r} , H_{4r} are

$$\begin{aligned} B_{2t} &= \frac{\theta_{t1} + \theta_{s2}}{\theta_{t2}} B_{2r} - c_0 \frac{\theta_{s2}}{\theta_{t2}} \mu_0 H_{2r} \\ B_{4t} &= \frac{\theta_{t4} + \theta_{s4}}{\theta_{t4}} B_{4r} - c_0 \frac{\theta_{s4}}{\theta_{t4}} \mu_0 H_{4r} \end{aligned} \quad (5.80)$$

The empirical coefficient c_0 takes into account the tooth magnetic unloading due to the slot flux density contribution.

Finally, the computing algorithm is shown on Figure 5.14 and starts with initial equivalent permeabilities.

For the next cycle of computation, each permeability is changed according to

$$\mu^{(i+1)} = \mu^{(i)} + c_1 (\mu^{(i+1)} - \mu^{(i)}) \quad (5.81)$$

It has been proved that $c_1 = 0.3$ is an adequate value, for a wide power range.

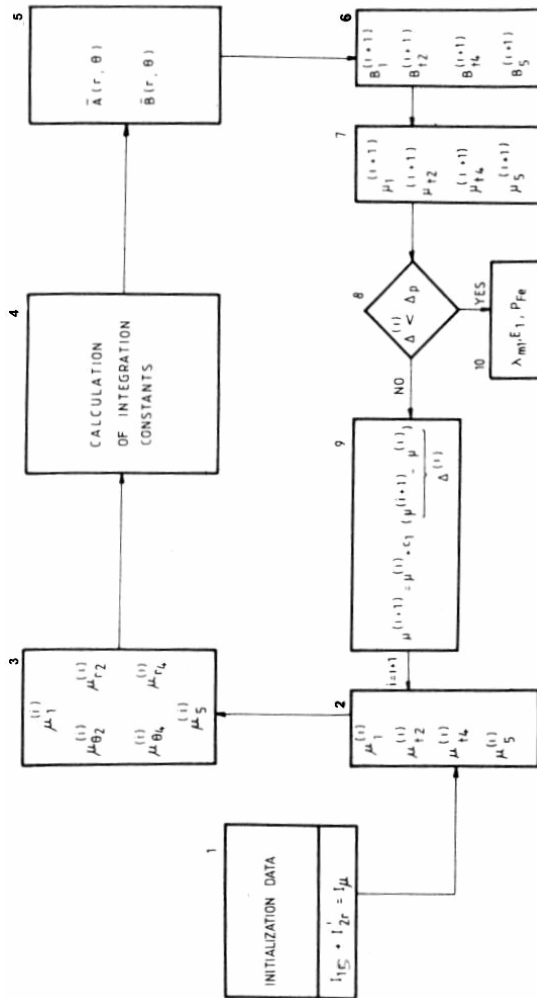


Figure 5.14 The computation algorithm

Model validation on no-load

The AIM has been applied to 12 different three-phase IMs from 0.75 kW to 15 kW (two-pole and four-pole motors), to calculate both the magnetization curve $I_{10} = f(E_1)$ and the core losses on no load for various voltage levels.

The magnetizing current is $I_{\mu} = I_{1r}$ and the no load active current I_{0A} is

$$I_{0A} = \frac{P_{Fe} + P_{mv} + P_{copper}}{3V_0} \quad (5.82)$$

The no-load current I_{10} is thus

$$I_{10} = \sqrt{I_{\mu}^2 + I_{0A}^2} \quad (5.83)$$

and

$$E_1 \approx V_0 - \omega_1 L_{1\sigma} I_{10} \quad (5.84)$$

where $L_{1\sigma}$ is the stator leakage inductance (known). Complete expressions of leakage inductances are introduced in Chapter 6.

Figure 5.15 exhibits computation results obtained with the conventional nonlinear model, the proposed model, and experimental data.

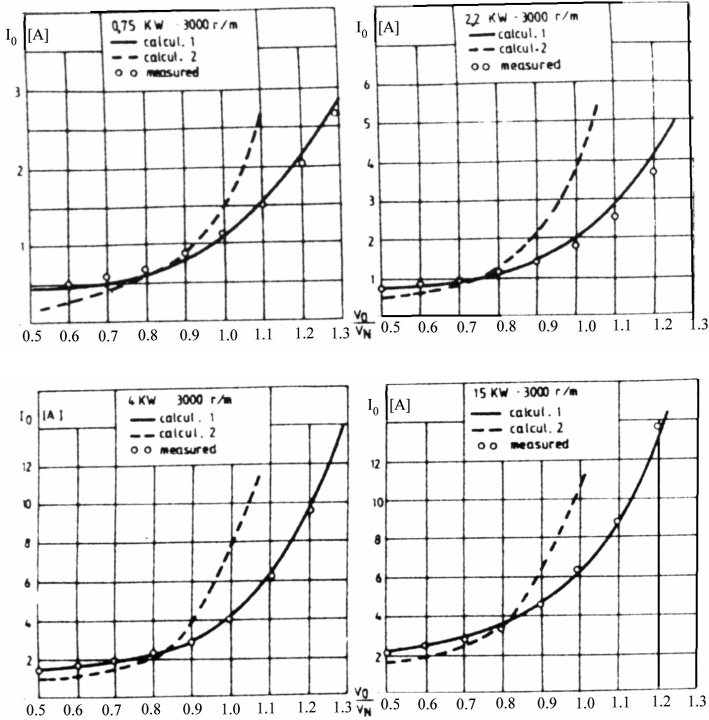


Figure 5.15 Magnetization characteristic validation on no load
1. conventional nonlinear model (paragraph 5.4.1); 2. AIM; 3. experiments

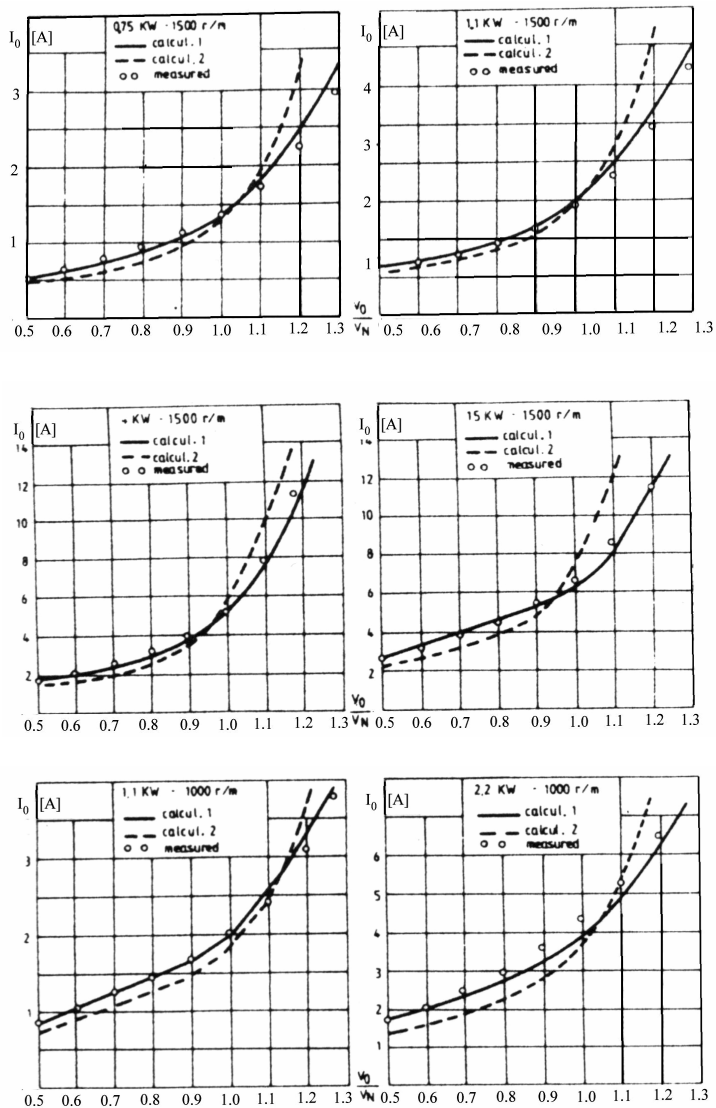


Figure 5.15 (continued)

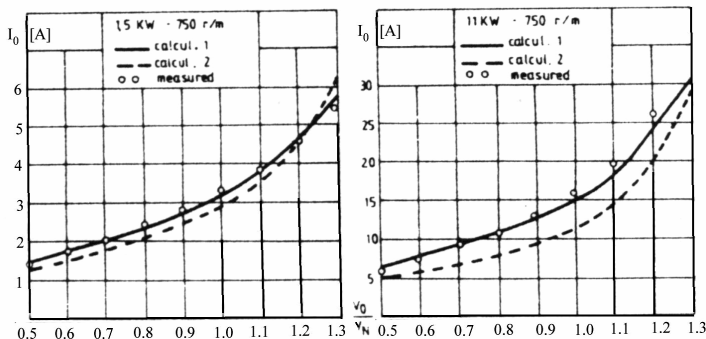


Figure 5.15 (continued)

It seems clear that AIM produces very good agreement with experiments even for high saturation levels.

A few remarks on AIM seem in order.

- The slotting is only globally accounted for by defining different tangential and radial permeabilities in the stator and rotor teeth.
- A single, but variable, permeability characterizes each of the machine domains (teeth and back cores).
- AIM is a bi-directional field approach and thus both radial and tangential flux density components are calculated.
- Provided the rotor equivalent current I_r (RMS value and phase shift with respect to stator current) is given, AIM allows calculation of the distribution of main flux in the machine on load.
- Heavy saturation levels ($V_0/V_n > 1$) are handled satisfactorily by AIM.
- By skewing the stack axially, the effect of skewing on main flux distribution can be handled.
- The computation effort is minimal (a few seconds per run on a contemporary PC).

So far AIM was used considering that the spatial field distribution is sinusoidal along stator bore. In reality, it may depart from this situation as shown in the previous paragraph. We may repeatedly use AIM to produce the actual spatial flux distribution or the airgap flux harmonics.

AIM may be used to calculate saturation-caused harmonics. The total mmf F_{1m} is still considered sinusoidal (Figure 5.16).

The maximum airgap flux density B_{g1m} (sinusoidal in nature) is considered known by using AIM for given stator (and eventually also rotor) current RMS values and phase shifts.

By repeatedly using the Ampere's law on contours such as those in Figure 5.17 at different position θ , we may find the actual distribution of airgap flux density, by admitting that the tangential flux density in the back core retains the

sinusoidal distribution along θ . This assumption is not, in general, far from reality as was shown in paragraph 5.4.3.

Ampere's law on contour Γ (Figure 5.17) is

$$\oint_{\Gamma} \overline{H} d\overline{l} = \int_{-p_1\theta}^{p_1\theta} F_{lm} \sin p_1\theta d(p_1\theta) = 2F_{lm} \cos p_1\theta \quad (5.85)$$

The left side of (5.85) may be broken into various parts (Figure 5.17). It may easily be shown that, due to the absence of any mmf within contours $ABB'B''A''A'A$ and $DCC''D''$,

$$\int_{ABB'A''B''A''} \overline{H} d\overline{l} = \int_{AA''} \overline{H} d\overline{l}; \quad \int_{CDD''C''} \overline{H} d\overline{l} = \int_{CC''} \overline{H} d\overline{l} \quad (5.86)$$

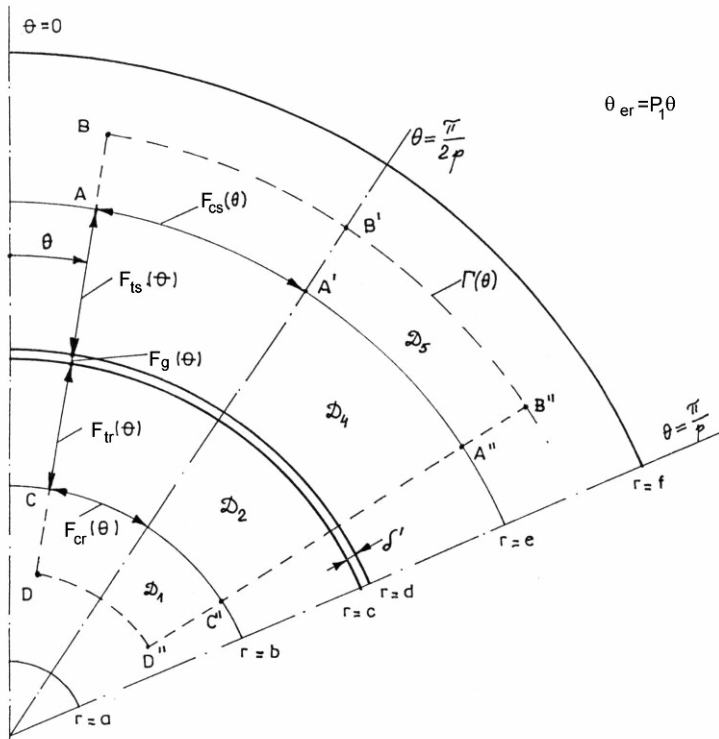


Figure 5.16 Ampere's law contours

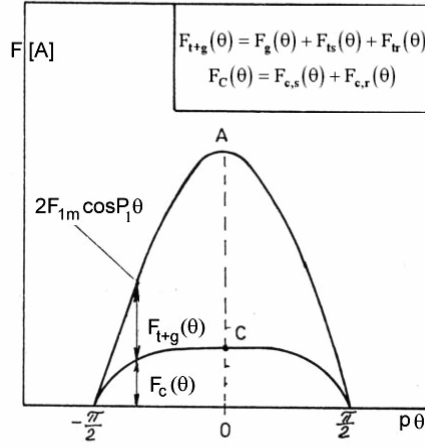


Figure 5.17 Mmf-total and back core component F

We may now divide the mmf components into two categories: those that depend directly on the airgap flux density $B_g(\theta)$ and those that do not (Figure 5.17).

$$2F_{t+g} = 2F_g(\theta) + 2F_{ts}(\theta) + 2F_{tr}(\theta) \quad (5.87)$$

$$F_c(\theta) = F_{cs}(\theta) + F_{cr}(\theta) \quad (5.88)$$

Also, the airgap mmf $F_g(\theta)$ is

$$F_g(\theta) = \frac{gK_c}{\mu_0} B_g(\theta) \quad (5.89)$$

Suppose we know the maximum value of the stator core flux density, for given F_{1m} , as obtained from AIM, B_{csm} ,

$$B_{cs}(\theta) = B_{csm} \sin p_1 \theta \quad (5.90)$$

We may now calculate $F_{cs}(\theta)$ as

$$F_{cs}(\theta) = 2 \int_{p_1 \theta}^{\pi/2} [H_{cs}(B_{csm} \sin \alpha)] d\alpha \quad (5.91)$$

The rotor core maximum flux density B_{crm} is also known from AIM for the same mmf F_{1m} . The rotor core mmf $F_{cr}(\theta)$ is thus

$$F_{cr}(\theta) = 2 \int_{p_1 \theta}^{\pi/2} [H_{cr}(B_{crm} \sin \alpha)] b d\alpha \quad (5.92)$$

From (5.88) $F_c(\theta)$ is obtained. Consequently,

$$F_{t+g} = 2F_{lm} \cos p\theta - F_{cr}(\theta) - F_{cs}(\theta) \quad (5.93)$$

Now in (5.87) we have only to express $F_{ts}(\theta)$ and $F_{tr}(\theta)$ —tooth mmfs—as functions of airgap flux density $B_g(\theta)$. This is straightforward.

$$B_{ts}(\theta) = \frac{\beta_s}{\xi_s} B_g(\theta), \quad \xi_s = \left(\frac{\tau_s}{b_{ts}} \right)^{-1} \quad (5.94)$$

τ_s —stator slot pitch, b_{ts} —stator tooth, $\beta_s < 1$ —ratio of real and apparent tooth flux density (due to the flux deviation through slot).

Similarly, for the rotor,

$$B_{tr}(\theta) = \frac{\beta_r}{\xi_r} B_g(\theta), \quad \xi_r = \left(\frac{\tau_r}{b_{tr}} \right)^{-1} \quad (5.95)$$

$$\text{Now} \quad F_{t+g}(\theta) = 2B_{g0}(\theta) \frac{g}{\mu_0} + 2h_s H_{ts} \left[\frac{\beta_s}{\xi_s} B_g(\theta) \right] + 2h_r H_{tr} \left[\frac{\beta_r}{\xi_r} B_g(\theta) \right] \quad (5.96)$$

In (5.96) $F_{t+g}(\theta)$ is known for given θ from (5.93). $B_{g0}(\theta)$ is assigned an initial value in (5.96) and then, based on the lamination B/H curve, H_{ts} and H_{tr} are calculated. Based on this, F_{t+g} is recalculated and compared to the value from (5.93). The calculation cycle is reiterated until sufficient convergence is reached.

The same operation is performed for 20 to 50 values of θ per half-pole to obtain smooth $B_g(\theta)$, $B_{ts}(\theta)$, $B_{tr}(\theta)$ curves per given stator (and rotor) mmfs (currents). Fourier analysis is used to calculate the harmonics contents. Sample results concerning the airgap flux density distribution for three designs of the same motor are given in Figures 5.18 through 5.21. [8]

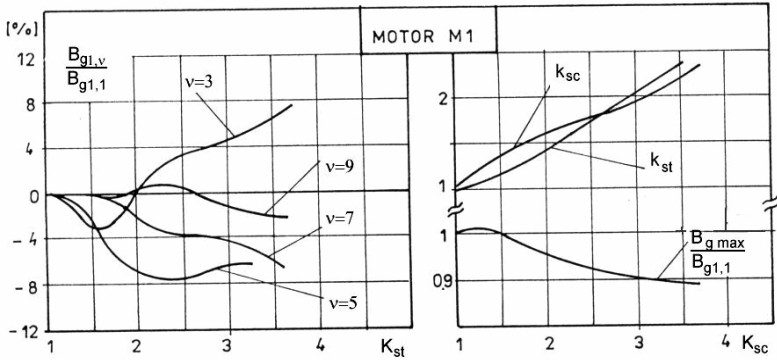


Figure 5.18 IM with equally saturated teeth and back cores

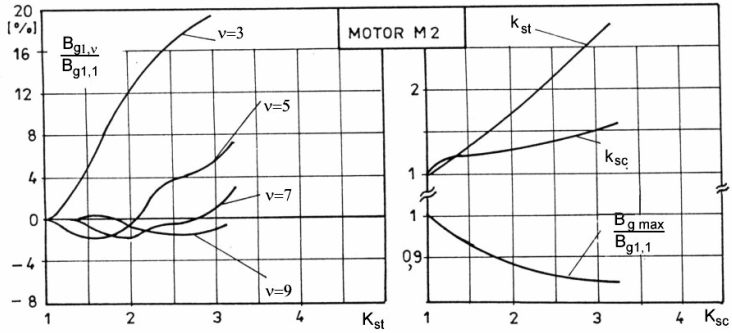


Figure 5.19 IM with saturated teeth ($K_{st} > K_{sc}$)

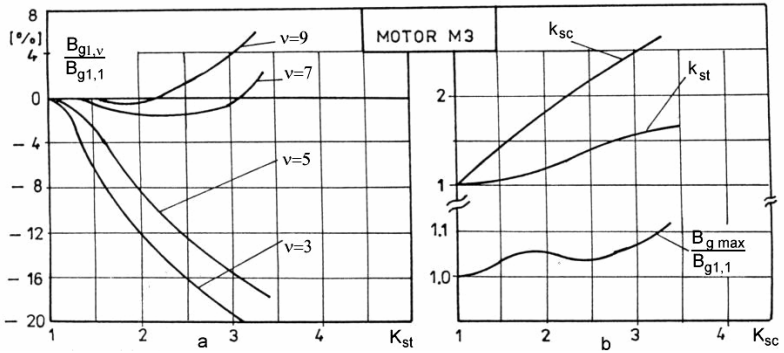


Figure 5.20 IM with saturated back cores

a.) airgap flux density b.) saturation factors K_{st} , K_{sc} , versus total saturation factor K_s

The geometrical data are

M1—optimized motor = equal saturation of teeth and cores: $a = 0.015$ m, $b = 0.036$ m, $c = 0.0505$ m, $d = 0.051$ m, $e = 0.066$ m, $f = 0.091$ m, $\xi_s = 0.54$, $\xi_r = 0.6$

M2—oversaturated stator teeth ($K_{st} > K_{sc}$): $b = 0.04$ m, $e = 0.06$ m, $\xi_s = 0.42$, $\xi_r = 0.48$

M3—oversaturated back cores ($K_{sc} > K_{st}$): $b = 0.033$ m, $e = 0.069$ m, $\xi_s = 0.6$, $\xi_r = 0.65$

The results on Figure 5.18 – 5.21 may be interpreted as follows:

- If the saturation level (factor) in the teeth and back cores is about the same (motor M1 on Figure 5.21), then even for large degrees of saturation ($K_s = 2.6$ on Figure 5.18b), the airgap flux harmonics are small (Figure 5.18a). This is why a design may be called optimized.

- If the teeth and back core saturation levels are notably different (oversaturated teeth is common), then the harmonics content of airgap flux density is rich. Flattened airgap flux distribution is obtained for oversaturated back cores (Figure 5.20).

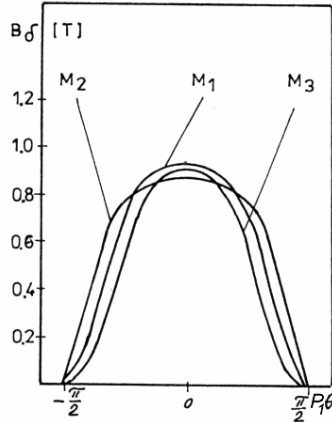


Figure 5.21 Airgap flux density distribution for three IM designs, M1, M2, M3.

- The above remarks seem to suggest that equal saturation level in teeth and back iron is to be provided to reduce airgap flux harmonics content. However, if core loss minimization is aimed at, this might not be the case, as discussed in Chapter 11 on core losses.

5.5 THE EMF IN AN A.C. WINDING

As already shown, the placement of coils in slots, the slot openings, and saturation cause airgap flux density harmonics (Figure 5.22).

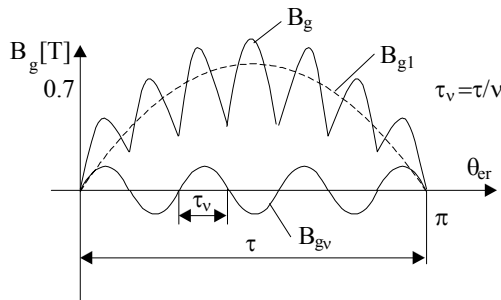


Figure 5.22 Airgap flux density and harmonics v

The voltage (emf) induced in the stator and rotor phases (bars) is of paramount importance for the IM behavior. We will derive its general

expression based on the flux per pole (Φ_v) for the v harmonic ($v = 1$ for the fundamental here; it is an “electrical” harmonic).

$$\Phi_v = \frac{2}{\pi} \tau_v L_e B_{gv} \quad (5.97)$$

For low and medium power IMs, the rotor slots are skewed by a distance c (in meters here). Consequently, the phase angle of the airgap flux density along the inclined rotor (stator) slot varies continuously (Figure 5.23).

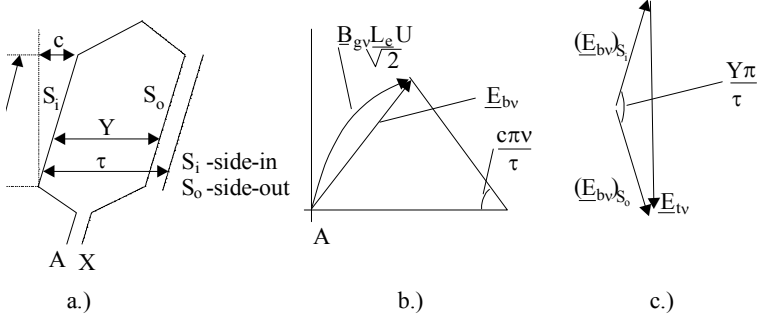


Figure 5.23 Chorded coil in skewed slots
a.) coil b.) emf per conductor c.) emf per turn

Consequently, the skewing factor $K_{\text{skew}v}$ is (Figure 5.23b)

$$K_{\text{skew}v} = \frac{\overline{AB}}{\overset{\frown}{AOB}} = \frac{\sin \frac{c}{\tau} v \frac{\pi}{2}}{\frac{c}{\tau} v \frac{\pi}{2}} \leq 1 \quad (5.98)$$

The emf induced in a bar (conductor) placed in a skewed slot E_{bv} (RMS) is

$$E_{bv} = \frac{1}{2\sqrt{2}} \omega_1 K_{\text{skew}v} \Phi_v \quad (5.99)$$

with ω_1 —the primary frequency (time harmonics are not considered here).

As a turn may have chorded throw (Figure 5.23a), the emfs in the two turn sides S_i and S_o (in Figure 5.23c) are dephased by less than 180° . A vector composition is required and thus E_{tv} is

$$E_{tv} = 2K_{yv} E_{bv}; \quad K_{yv} = \sin v \frac{Y}{2} \frac{\pi}{2} \leq 1 \quad (5.100)$$

K_{yv} is the chording factor.

There are n_c turns per coil, so the coil emf E_{cv} is

$$E_{cv} = n_c E_{tv} \quad (5.101)$$

In all there are q neighbouring coils per pole per phase. Let us consider them in series and with their emfs phase shifted by $\alpha_{ecv} = 2\pi p_1 v / N_s$ (Figure 5.24). The resultant emf for the q coils (in series in our case) E_{cv} is

$$E_{cv} = q E_{cv} K_{qv} \quad (5.102)$$

K_{qv} is called the distribution factor. For an integer q (Figure 5.24) K_{qv} is

$$K_{qv} = \frac{\sin v\pi / 6}{q \sin(v\pi / 6q)} \leq 1 \quad (5.103)$$

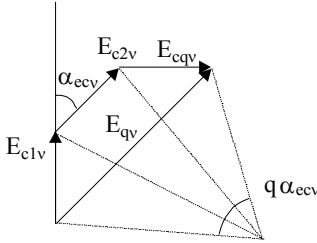


Figure 5.24 Distribution factor K_{qv}

The total number of q coil groups is p_1 (pole pairs) in single layer and $2p_1$ in double layer windings. Now we may introduce $a \leq p_1$ current paths in parallel.

The total number of turns/phase is $W_1 = p_1 q n_c$ for single layer and, $W_1 = 2p_1 q n_c$ for double layer windings, respectively. Per current path, we do have W_a turns.

$$W_a = \frac{W_1}{a} \quad (5.104)$$

Consequently the emf per current path E_a is

$$E_{av} = \pi \sqrt{2} f_1 W_a K_{qv} K_{yv} K_{cv} \Phi_v \quad (5.105)$$

For the stator phase emf, when the stator slots are straight, $K_{cv} = 1$.

For the emf induced in the rotor phase (bar) with inclined rotor slots, $K_{cv} \neq 1$ and is calculated with (5.98).

$$(E_{av})_{\text{stator}} = \pi \sqrt{2} f_1 W_a K_{qv}^s K_{yv}^s \Phi_v \quad (5.106)$$

$$(E_{bv})_{\text{rotor}} = \frac{\pi}{\sqrt{2}} f_1 K_{skewv} \Phi_v \quad (5.107)$$

$$(E_{av})_{\text{rotor}} = \pi \sqrt{2} f_1 W_a K_{qv}^r K_{yv}^r \Phi_v \quad (5.108)$$

The distribution K_{qv} and chording (K_{yv}) factors may differ in the wound rotor from stator.

Let us note that the distribution, chording, and skewing factors are identical to those derived for mmf harmonics. This is so because the source of airgap

field distribution is the mmf distribution. The slot openings, or magnetic saturation influence on field harmonics is lumped into Φ_v .

The harmonic attenuation or cancellation through chording or skewing is also evident in (5.102) – (5.104). Harmonics that may be reduced or cancelled by chording or skewing are of the order

$$v = Km \pm 1 \quad (5.109)$$

For one of them

$$v \frac{y}{\tau} \frac{\pi}{2} = K' \pi \quad \text{or} \quad v \frac{c}{\tau} \frac{\pi}{2} = K'' \pi \quad (5.110)$$

These are called mmf step (belt) harmonics (5, 7, 11, 13, 17, 19). Third mmf harmonics do not exist for star connection. However, as shown earlier, magnetic saturation may cause 3rd order harmonics, even 5th, 7th, 11th order harmonics, which may increase or decrease the corresponding mmf step harmonics.

Even order harmonics may occur in two-phase windings or in three-phase windings with fractionary q . Finally, the slot harmonics orders v_c are:

$$v_c = 2Kqm \pm 1 \quad (5.111)$$

For fractionary $q = a + b/c_1$,

$$v_c = 2(ac_1 + b) \left(\frac{K}{c_1} \right) m \pm 1 \quad (5.112)$$

We should notice that for all slot harmonics,

$$K_{qv} = \frac{\sin(2Kqm \pm 1)\pi/6}{q \sin[(2Kqm \pm 1)\pi/6q]} = K_{q1} \quad (5.113)$$

Slot harmonics are related to slot openings presence or to the corresponding airgap permeance modulation. All slot harmonics have the same distribution factor with the fundamental, they may not be destroyed or reduced. However, as their amplitude (in airgap flux density) decreases with v_c increasing, the only thing we may do is to increase their lowest order. Here the fractionary windings come into play (5.112). With q increased from 2 to 5/2, the lowest slot integer harmonic is increased (see (5.111) – (5.112)) from $v_{cmin} = 11$ to $v_{cmin} = 29$! ($K/c_1 = 2/2$). As K is increased, the order v of step (phase belt) harmonics (5.111) may become equal to v_c .

Especially the first slot harmonics ($K = 1$, $K/c_1 = 1$) may thus interact and amplify (or attenuate) the effect of mmf step harmonics of the same order. Intricate aspects like this will be dealt with in Chapter 10.

5.6 THE MAGNETIZATION INDUCTANCE

Let us consider first a sinusoidal airgap flux density in the airgap, at no load (zero rotor currents). Making use of Ampere's law we get, for airgap flux amplitude,

$$B_{g1} \approx \frac{\mu_0 (F_1)_{\text{phase}}}{g K_c K_s}; \quad K_s > 1 \quad (5.114)$$

where F_1 is the phase mmf/pole amplitude,

$$(F_1)_{\text{phase}} = \frac{2 W_1 I_0 \sqrt{2} K_{q1} K_{y1}}{\pi p_1} \quad (5.115)$$

For a three-phase machine on no load,

$$F_{1m} = \frac{3 W_1 I_0 \sqrt{2} K_{q1} K_{y1}}{\pi p_1} \quad (5.116)$$

Now the flux per pole, for the fundamental Φ_1 , is

$$\Phi_1 = \frac{2}{\pi} B_{g1} \tau L_e \quad (5.117)$$

So the airgap flux linkage per phase ($a = 1$) Ψ_{11h} is

$$\Psi_{11h} = W_1 K_{q1} K_{y1} \Phi_1 \quad (5.118)$$

The magnetization inductance of a phase, when all other phases on stator and rotor are open, L_{11m} is

$$L_{11m} = \frac{\Psi_{11h}}{I_0 \sqrt{2}} = \frac{4 \mu_0 (W_1 K_{q1} K_{y1})^2}{\pi^2} \frac{L_e \tau}{p_1 g K_c K_s} \quad (5.119)$$

K_s —total saturation factor.

In a similar manner, for a harmonic v , we obtain

$$L_{11mv} = \frac{4 \mu_0 (W_1 K_{qv} K_{yv})^2}{\pi^2} \frac{L_e \tau}{p_1 g K_c K_{sv}}; \quad K_{sv} \approx (1-1.2) \quad (5.120)$$

The saturation coefficient K_{sv} tends to be smaller than K_s , as the length of harmonics flux line in iron is usually shorter than for the fundamental.

Now, for three-phase supply we use the same rationale, but, in the (5.114) F_{1m} will replace $(F_1)_{\text{phase}}$.

$$L_{1mv} = \frac{6 \mu_0 (W_1 K_{qv} K_{yv})^2}{\pi^2 v^2} \frac{L_e \tau}{p_1 g K_c K_{sv}} \quad (5.121)$$

When the no-load current (I_0/I_n) increases, the saturation factor increases, so the magnetization inductance L_{1m} decreases (Figure 5.25).

$$L_{1m} = \frac{6\mu_0 (W_l K_{q1} K_{y1})^2}{\pi^2} \frac{L_e \tau}{p_l g K_c K_s (I_0 / I_n)} \quad (5.122)$$

In general, a normal inductance, L_n , is defined as

$$L_n = \frac{V_n}{I_n \omega_{1n}} : I_{1m} = \frac{L_{1m}}{L_n} \quad (5.123)$$

V_n , I_n , ω_{1n} rated phase voltage, current, and frequency of the respective machine.

Well designed induction motors have I_{1m} (Figure 5.25) in the interval 2.5 to 4; in general, it increases with power level and decreases with large number of poles.

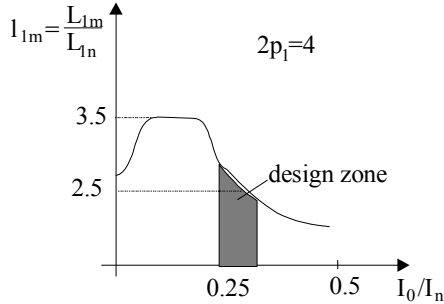


Figure 5.25 Magnetization inductance versus no load current I_0/I_n

It should be noticed that Figure 5.25 shows yet another way (scale) of representing the magnetization curve.

The magnetization inductance of the rotor L_{22m} has expressions similar to (5.119) – (5.120), but with corresponding number of turns phase, distribution, and chording factors.

However, for the mutual magnetising inductance between a stator and rotor phase L_{12m} , we obtain

$$L_{12mv} = \frac{4\mu_0}{\pi^2} \left(W_1 K_{qv}^s K_{yv}^s W_2 K_{qv}^r K_{yv}^r K_{cv}^r \right) \frac{L_e \tau}{p_l g K_{sv}} \quad (5.124)$$

This time, the skewing factor is present (skewed rotor slots).

From (5.124) and (5.120),

$$\frac{L_{12mv}}{L_{11mv}} = \frac{W_2 K_{qv}^r K_{yv}^r K_{cv}^r}{W_1 K_{qv}^s K_{yv}^s} \quad (5.125)$$

It is now evident that for a rotor cage, $K_{qv}^r = K_{yv}^r = 1$ as each bar (slot) may be considered as a separate phase with $W_2 = 1/2$ turns.

The inductance L_{11m} or L_{22m} are also called main (magnetization) self inductances, while L_{1m} is called the cyclic (multiphase supply) magnetizing inductance. In general, for m phases,

$$L_{1m} = \frac{m}{2} L_{11m}; \quad m = 3 \text{ for three phases} \quad (5.126)$$

Example 5.1. The magnetization inductance L_{1m} calculation

For the motor M1 geometry given in the previous paragraph, with double-layer winding and $N_s = 36$ slots, $N_r = 30$ slots, $2p_1 = 4$, $y/\tau = 8/9$, let us calculate L_{1m} and no-load phase current I_0 for a saturation factor $K_s = 2.6$ and $B_{g1} = 0.9T$, $K_{st} = K_{sr} = 1.8$, $K_c = 1.3$ (Figure 5.20, 5.21).

Stator (rotor) slot opening are: $b_{os} = 6g$, $b_{or} = 3g$, $W_1 = 120\text{turns/phase}$, $L_c = 0.12m$, and stator bore diameter $D_i/2 = 0.051m$.

Solution

- The Carter coefficient K_c ((5.4), (5.3)) is

$$K_{cl,2} = \frac{\tau_{s,r}}{\tau_{s,r} - \gamma_{1,2} \cdot g / 2} = \begin{cases} 1.280 \\ 1.0837 \end{cases} \quad (5.127)$$

$$\begin{aligned} \tau_s &= \frac{\pi D_i}{N_s} = \frac{\pi \cdot 2 \cdot 0.051}{36} = 9.59 \cdot 10^{-3} m \\ \tau_r &= \frac{\pi(D_i - 2g)}{N_r} = \frac{\pi(2 \cdot 0.051 - 2 \cdot 0.0005)}{30} = 10.57 \cdot 10^{-3} m \end{aligned} \quad (5.128)$$

$$\begin{aligned} \gamma_1 &= \frac{(2b_{os} / g)^2}{5 + 2b_{os} / g} = \frac{12^2}{5 + 12} = 8.47 \\ \gamma_2 &= \frac{(2b_{or} / g)^2}{5 + 2b_{or} / g} = \frac{6^2}{5 + 6} = 3.27 \end{aligned} \quad (5.129)$$

$$K_c = K_{c1} \cdot K_{c2} = 1.280 \cdot 1.0837 = 1.327 \quad (5.130)$$

- The distribution and chording factors K_{q1} , K_{y1} , (5.100), (5.103) are

$$K_{q1} = \frac{\sin \pi / 6}{3 \sin \pi / 6 \cdot 3} = 0.9598; \quad K_{y1} = \sin \frac{8}{9} \frac{\pi}{2} = 0.9848 \quad (5.131)$$

From (5.122),

$$L_{lm} = 6 \cdot \frac{1.25 \cdot 10^{-6}}{\pi^2} \cdot (300 \cdot 0.9598 \cdot 0.9848)^2 \cdot \frac{9 \cdot 9.59 \cdot 10^{-3} \cdot 0.12}{2 \cdot 0.5 \cdot 10^{-3} \cdot 1.327 \cdot 2.6} = 0.1711 \text{H} \quad (5.132)$$

- The no load current I_0 from (5.114) is

$$B_{gl} = \frac{\mu_0 F_{lm}}{g K_c K_s} = \frac{1.256 \cdot 10^{-3} \cdot 3\sqrt{2} \cdot 300 \cdot I_0 \cdot 0.9598 \cdot 0.9848}{3.14 \cdot 2 \cdot 0.5 \cdot 10^{-3} \cdot 1.327 \cdot 2.6} = 0.9 \text{T} \quad (5.133)$$

So the no load phase current I_0 is

$$I_0 = 3.12 \text{A} \quad (5.134)$$

Let us consider rated current $I_n = 2.5 I_0$.

- The emf per phase for $f_1 = 60 \text{Hz}$ is

$$E_1 = 2\pi f_1 L_{lm} I_0 = 2\pi 60 \cdot 0.1711 \cdot 3.12 = 201.17 \text{V} \quad (5.135)$$

With a rated voltage/phase $V_1 = 210 \text{ V}$, the relative value of L_{lm} is

$$I_{lm} = \frac{L_{lm}}{V_1 / (\omega_1 I_n)} = \frac{0.1711}{210 / (2\pi 60 \cdot 2.5 \cdot 3.12)} = 2.39 \quad (5.136)$$

It is obviously a small relative value, mainly due to the large value of saturation factor $K_s = 2.6$ (in the absence of saturation: $K_s = 1.0$).

In general, $E_1/V_1 \approx 0.9 - 0.98$; higher values correspond to larger power machines.

5.7 SUMMARY

- In absence of rotor currents, the stator winding currents produce an airgap flux density which contains a strong fundamental and spatial harmonics due to the placement of coils in slots (mmf harmonics), magnetic saturation, and slot opening presence. Essentially, FEM could produce a fully realistic solution to this problem.
- To simplify the study, a simplified analytical approach is conventionally used; only the mmf fundamental is considered.
- The effect of slotting is “removed” by increasing the actual airgap g to gK_c ($K_c > 1$); K_c —the Carter coefficient.
- The presence of eventual ventilating radial channels (ducts) is considered through a correction coefficient applied to the geometrical stack axial length.
- The dependence of peak airgap flux density on stator mmf amplitude (or stator current) for zero rotor currents, is called the magnetization curve.

- When the teeth are designed as the heavily saturated part, the airgap flux “flows” partially through the slots, thus “defluxing” to some the teeth extent.
- To account for heavily saturated teeth designs, the standard practice is to calculate the mmf $(F_1)_{30^\circ}$ required to produce the maximum (flat) airgap flux density and then increase its value to $F_{1m} = (F_1)_{30^\circ} / \cos 30^\circ$.
- A more elaborate two dimensional analytical iterative model (AIM), valid both for zero and nonzero rotor currents, is introduced to refine the results of the standard method. The slotting is considered indirectly through given but different radial and axial permeabilities in the slot zones. The results are validated on numerous low power motors, and sinusoidal airgap and core flux densities.
- Further on, AIM is used to calculate the actual airgap flux density distribution accounting for heavy magnetic saturation. For more saturated teeth, flat airgap flux density is obtained while, for heavier back core saturation, a peaked airgap and teeth flux density distribution is obtained.
- The presence of saturation harmonics is bound to influence the total core losses (as investigated in Chapter 11). It seems clear that if teeth and back iron cores are heavily but equally saturated, the flux density is still sinusoidal all along stator bore. The stator connection is also to be considered as for star connection, the stator no-load current is sinusoidal, and flux third harmonics may occur, while for the delta connection, the opposite is true.
- The expression of emf in a.c. windings exhibits the distribution, chording, and skewing factors already derived for mmfs in Chapter 4.
- The emf harmonics include mmf space (step) harmonics, saturation-caused space harmonics, and slot opening (airgap permeance variation) harmonics v_c .
- Slot flux (emf) harmonics v_c show a distribution factor K_{qv} equal to that of the fundamental ($K_{qv} = K_{q1}$) so they cannot be destroyed. They may be attenuated by increasing the order of first slot harmonics $v_{cmin} = 2qm - 1$ with larger q slots per pole per phase or/and by increased airgap, or by fractional q .
- The magnetization inductance L_{1m} valid for the fundamental is the ratio of phase emf, to angular stator frequency to stator current. L_{1m} is decreasing with airgap, number of pole pairs, and saturation level (factor), but it is proportional to pole pitch and stack length and equivalent number of turns per phase squared.
- In relative values, L_{1m} increases with motor power and decreases with larger number of poles, in general.

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